Taming turbulence via spectral nudging

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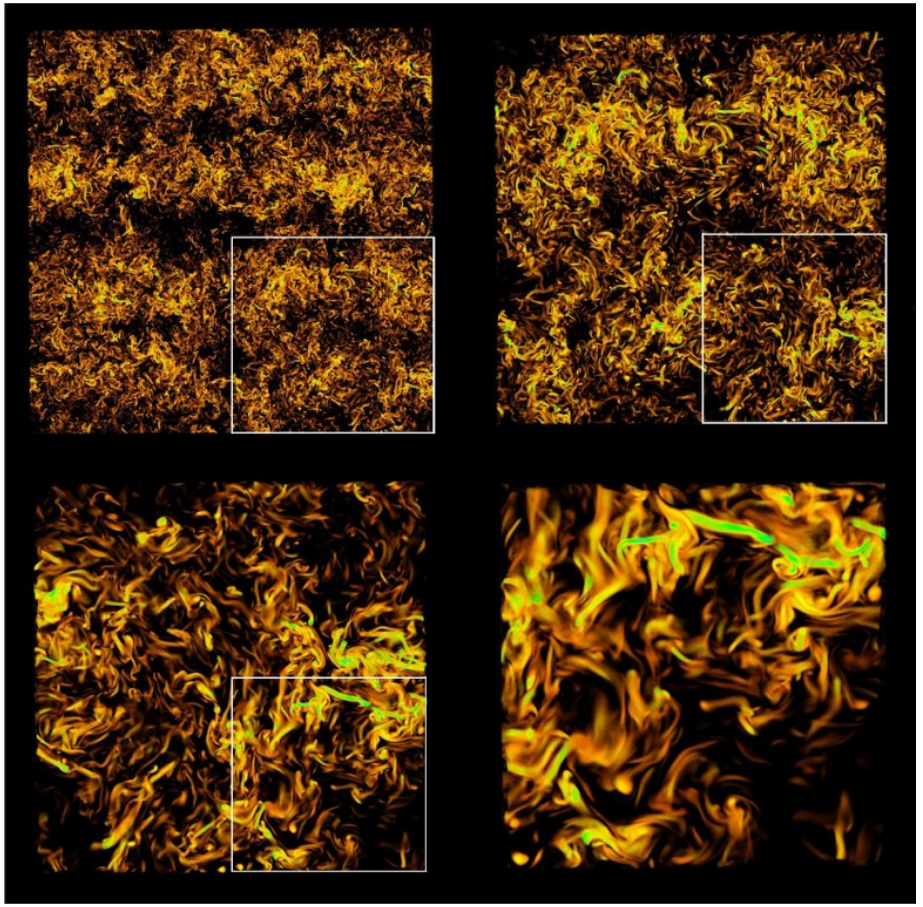
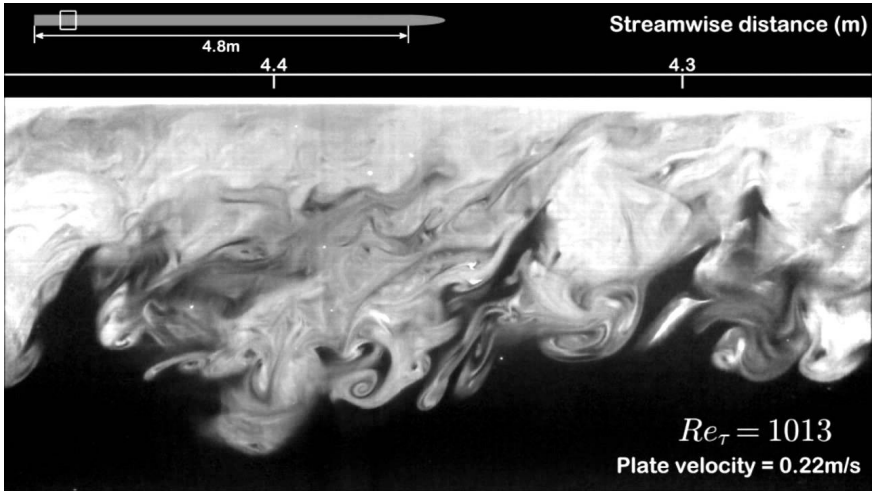
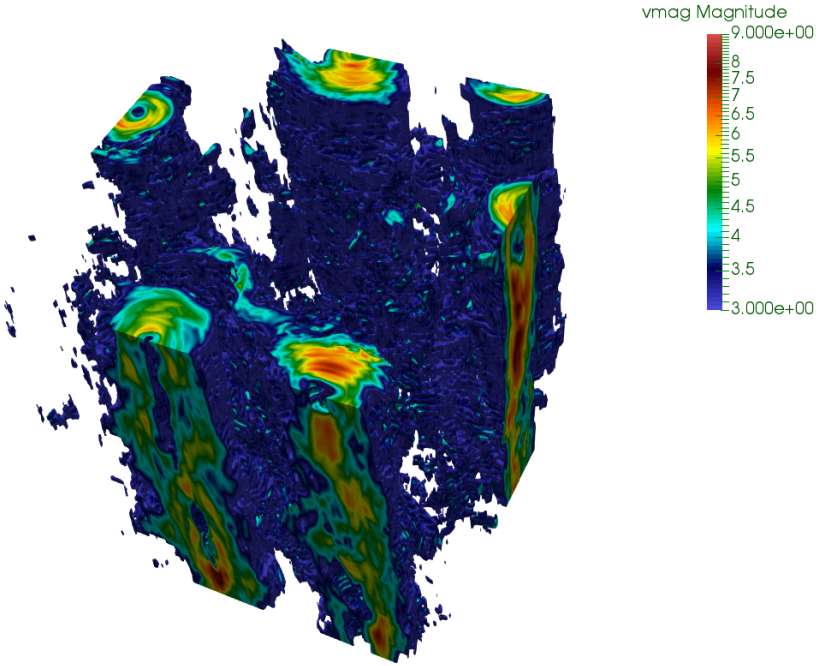
The keys of turbulence







What are the most relevant and flow-determining structures in a turbulence?

Let’s try to use nudging to find out!

What is nudging?

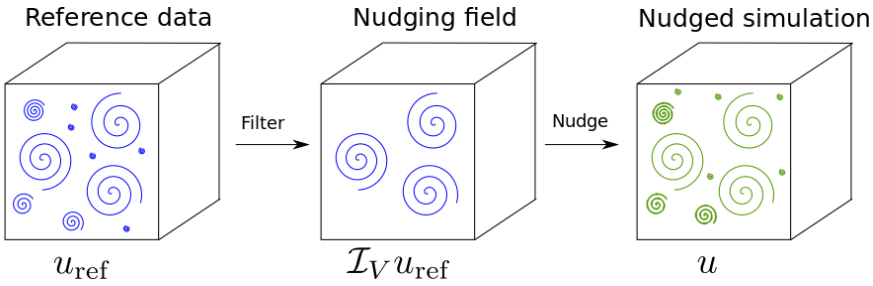
A method to control the evolution of a flow by incorporating reference data

Nudged simulation

Navier-Stokes plus a penalty term

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \nu \nabla^2 u - \alpha \mathcal{I}_V (u - u_{\text{ref}})$$

The term only acts where data is present



Reference data

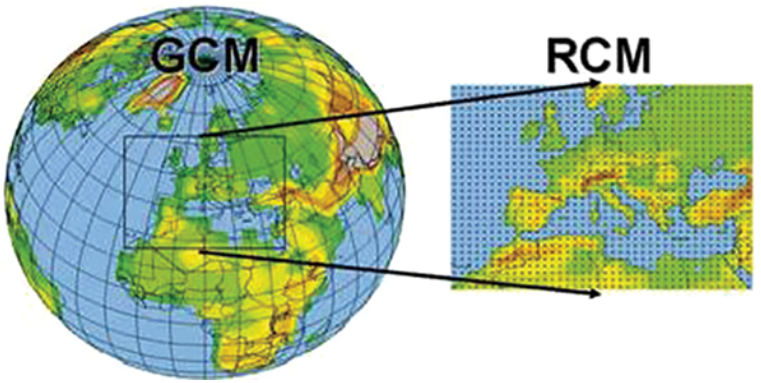
While this would traditionally come from measurements or observations, we can generate our “true” data by performing simulations of the Navier-Stokes equations

$$\frac{\partial u_{\text{ref}}}{\partial t} + (u_{\text{ref}} \cdot \nabla)u_{\text{ref}} = -\nabla p + \nu \nabla^2 u_{\text{ref}} + f$$

No linearization, or quasi-guassianity assumptions present!

What is nudging?

- It is a *Data Assimilation* technique that can be used to control the evolution of a chaotic flow and improve its predictability [Lakshmivarahan & Lewis (2013)]
- It is also used to incorporate data coming from a Global Circulation Model into a Regional Climate Model [Waldron et al. (1996)]



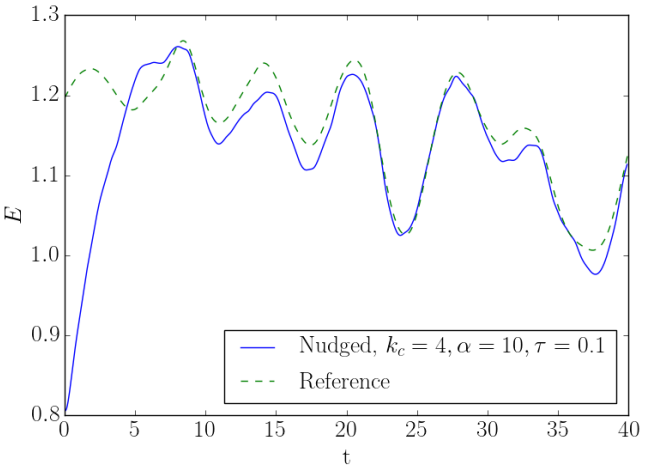
- Can we use nudging to probe the Eqs of motion?
- Is nudging strong enough to tame a 3D turbulent flow?
- What can we learn about turbulence in the process?

We will answer these questions using spectral nudging
(i.e. the filter is a low-pass Fourier filter)

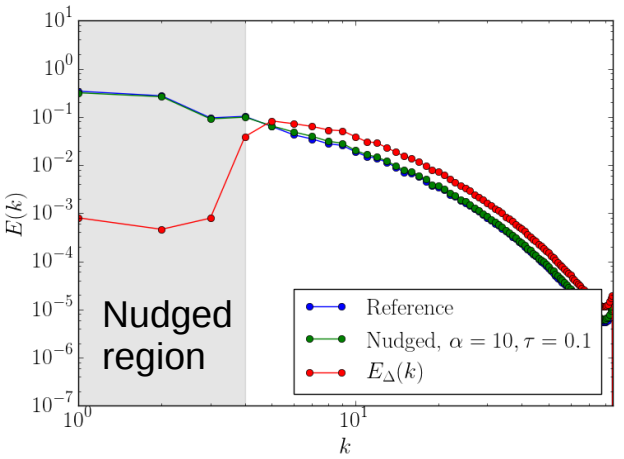
Comparing the simulations

In our numerical set-up we can access the smaller scales of the reference flow. This way we can compare with the “truth”.

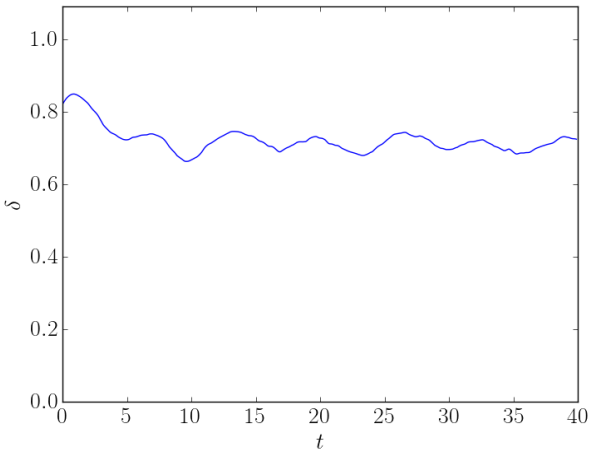
Energy



Energy spectra of differences



Velocity correlations



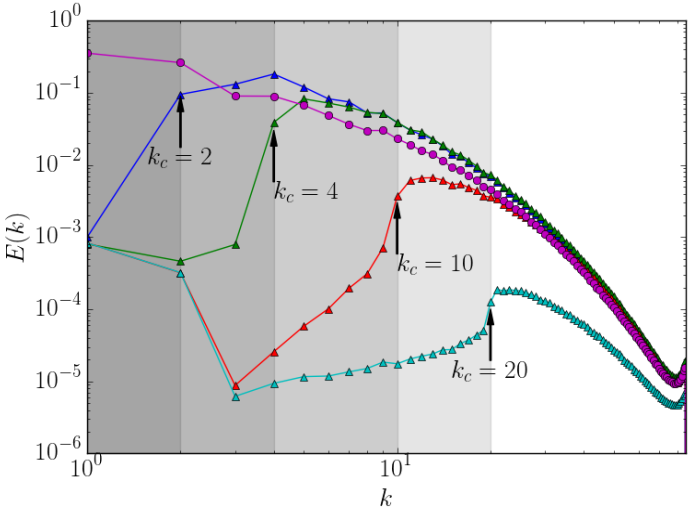
$$E_\Delta(k) = \sum_{|k|=k} |\hat{u} - \hat{u}_{\mathrm{ref}}|^2(k)$$

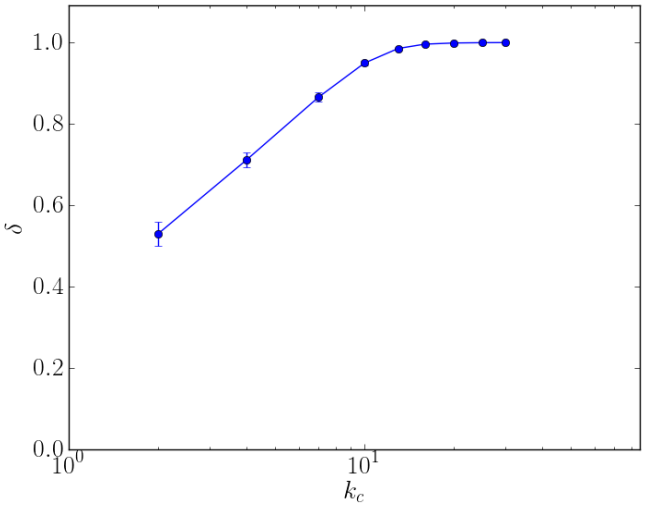
$$\delta = \frac{\langle u \cdot u_{\mathrm{ref}} \rangle}{\sqrt{\langle u^2 \rangle \langle u_{\mathrm{ref}}^2 \rangle}}$$

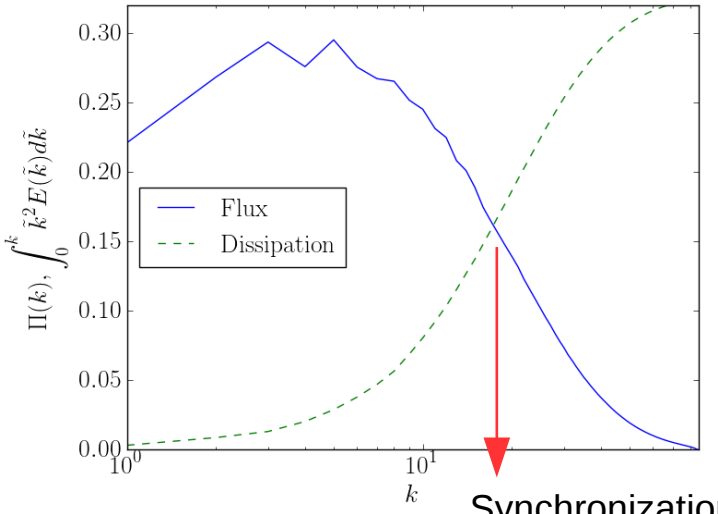
How much should one nudge a turbulent flow in order to control (synchronize) it?

We now perform a scan in the nudging wavenumber k_c using 3D turbulence as reference

Nudging at different scales







Synchronization scale

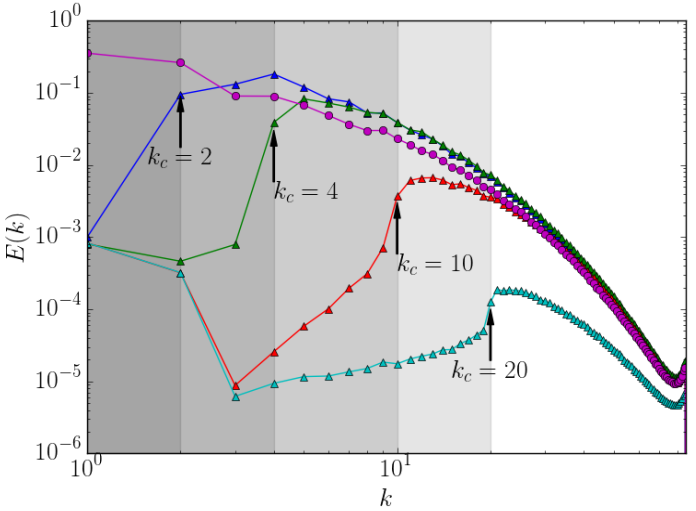
In order to achieve good correlation in the large and the small scales one needs to nudge up to the scales when dissipation starts becoming important

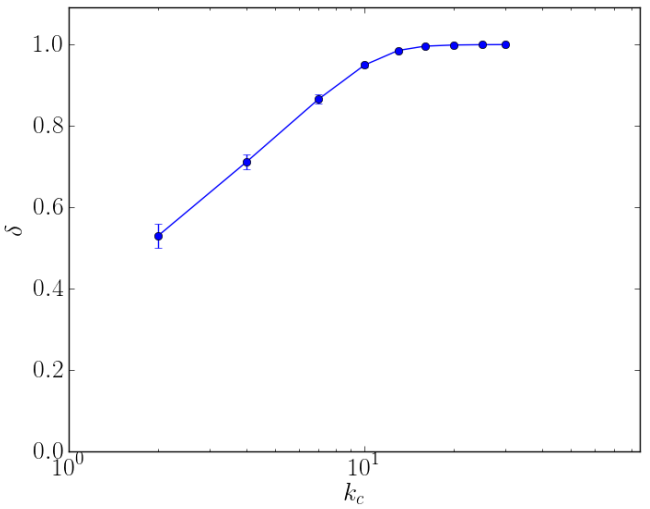
First systematic study in 3D turbulent flows

Only 1% of the modes are nudged!

$$\left(\frac{17}{81}\right)^3 \approx 0.01$$

Nudging at different scales





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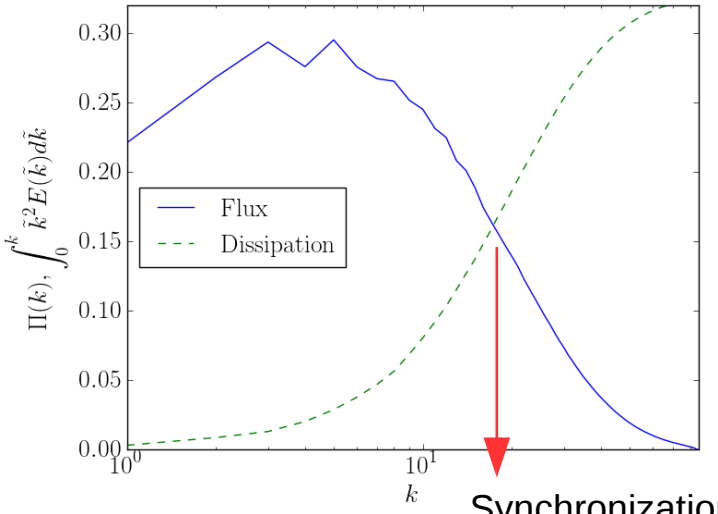
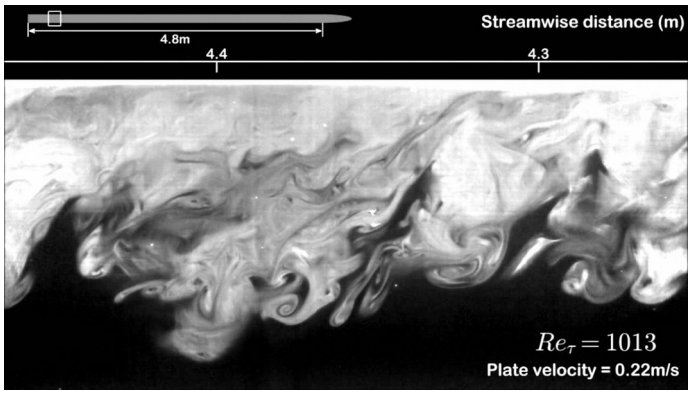


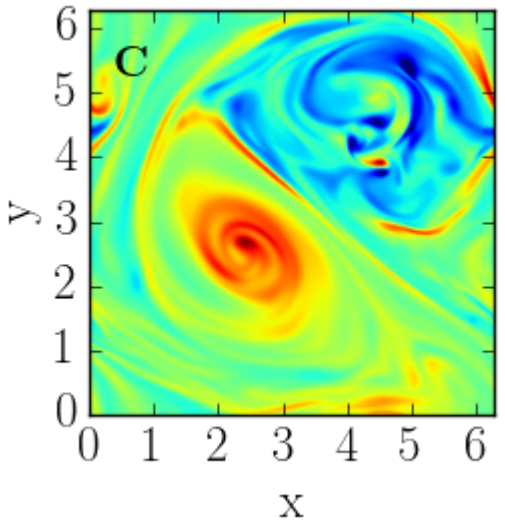
Synchronization scale



But how much or where to nudge is not so clear in non-homogenous flows!

Spectral nudging as a physics based parameter estimator

$\Omega = ???$



Given data with unknown parameters, can we use nudging to impose correlations and find out these values?

We put this idea to test with a rotating turbulence flow:

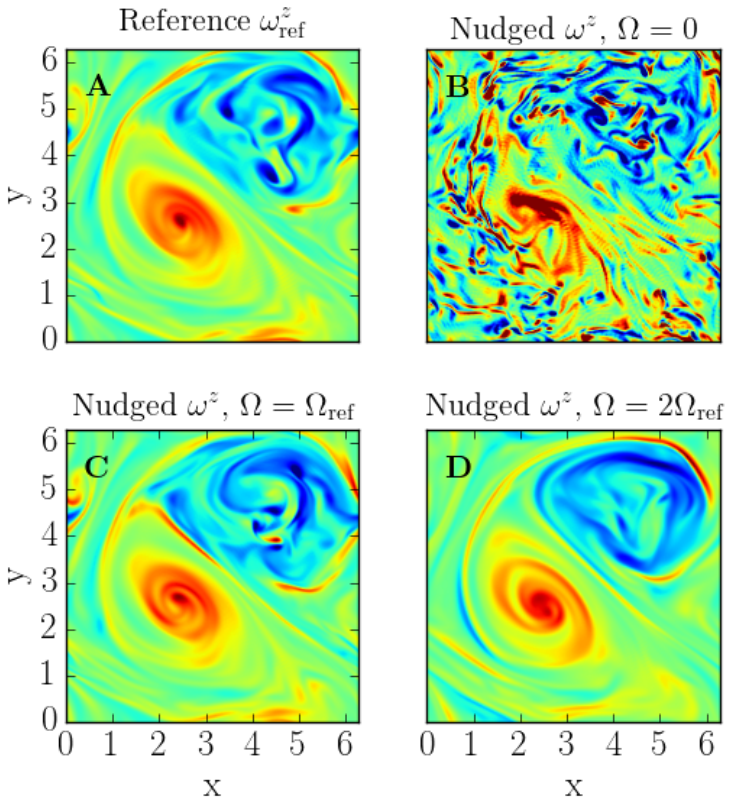
$$\frac{\partial u_{\mathrm{ref}}}{\partial t} + (u_{\mathrm{ref}} \cdot \nabla) u_{\mathrm{ref}} + 2\Omega_0 \times u_{\mathrm{ref}} = -\nabla p + \nu \nabla^2 u_{\mathrm{ref}} + f$$

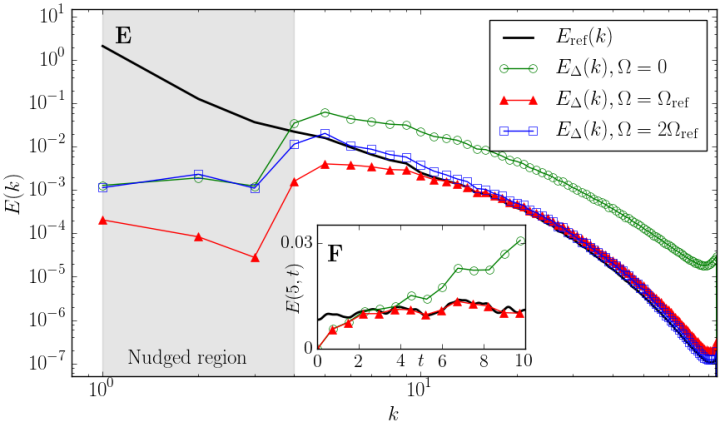
We can run different nudged simulations varying the rotation frequency Ω

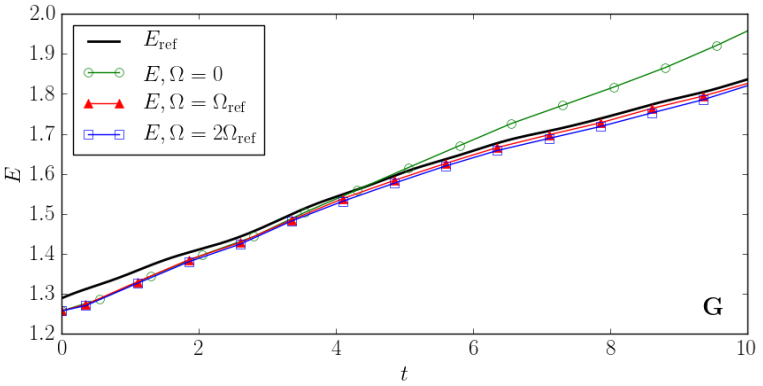
[Clark Di Leoni, Mazzino & Biferale, arXiv:1804.07680 (2018), just accepted in Physical Review Fluids]

Finding out parameters

Nudged simulations done varying the rotation frequency Ω
The reference has a frequency of Ω_0





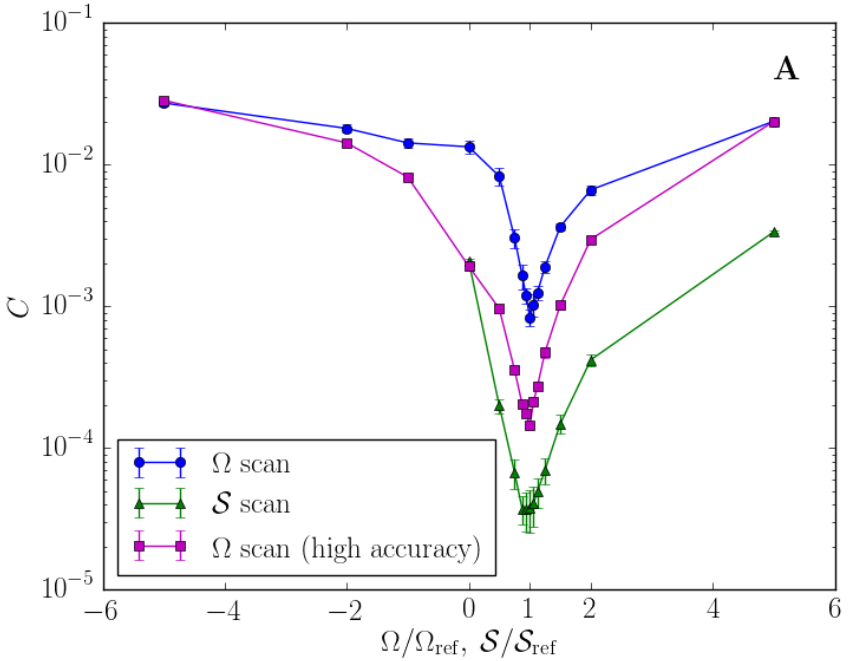


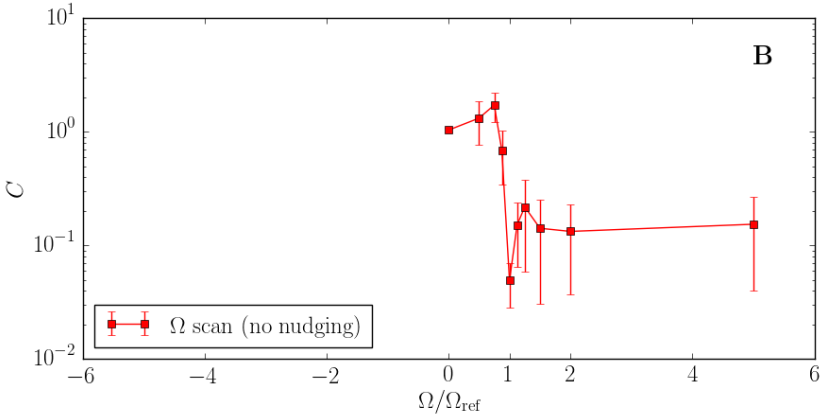
Nudging is sensitive to the choice of parameters!
It is easier to impose correlations on the correct equations!

Quantifying the search

$$C = \int_0^{k_c} E_\Delta(k) dk$$

Integrate only over the scales at which we have information

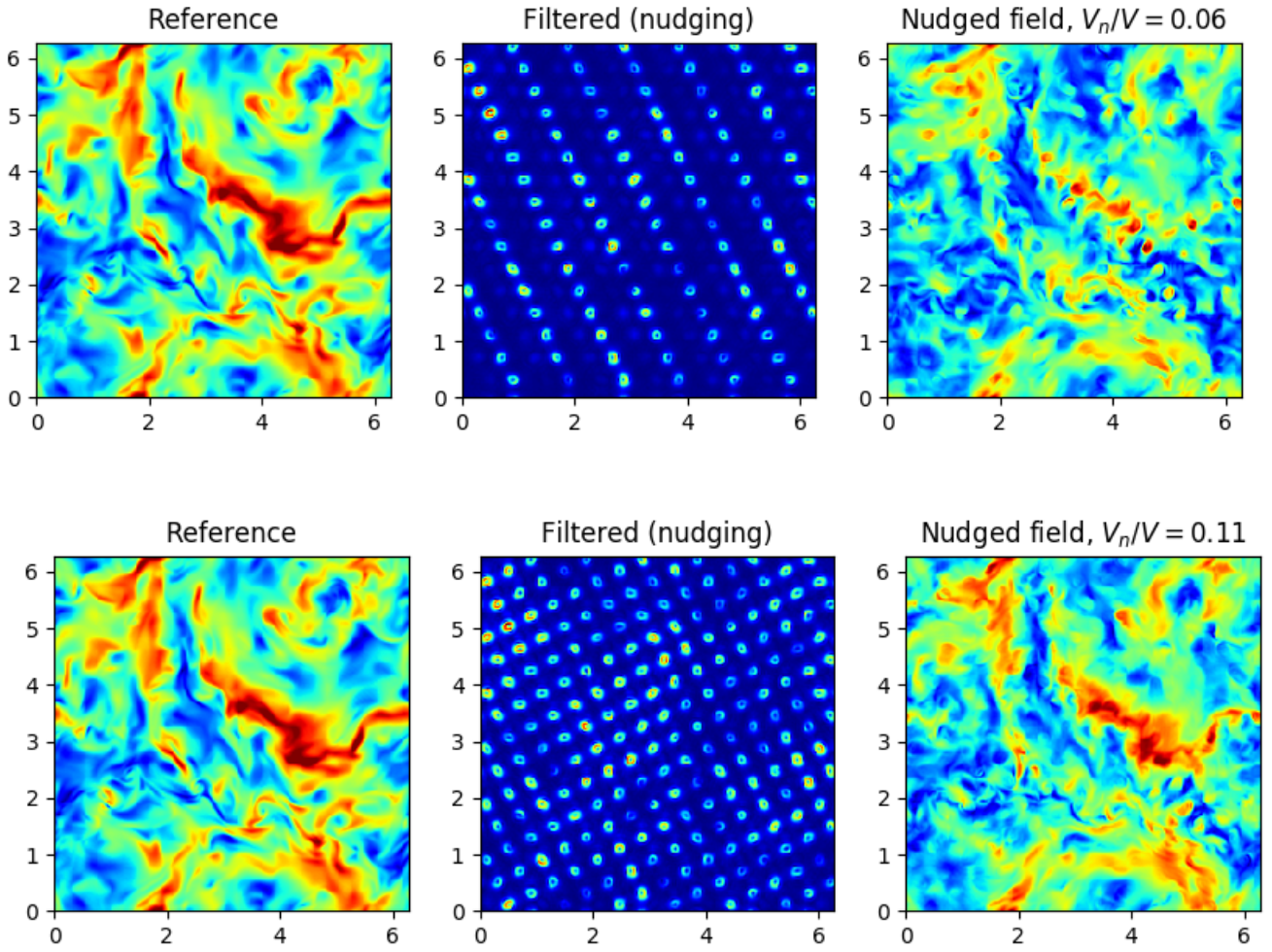




Scan with no nudging (just same initial condition)

- Nudging can be used to determine the values of different parameters.
- Errors between 3% and 10%!
- It's cheap, versatile and easy generalizable.
- No need to make assumptions such as linearity or quasi-Gaussianity.

Upcoming: Nudging in real space



How large a volume do we have to nudge in order to synchronize?
Can we identify key regions in real space?

Conclusions

- We explored how the spectral nudging technique can be applied to fully developed three dimensional turbulence
- Multiscale turbulent flows can be completely synchronized. It requires information requires information on a lot of scales, but not on all!
- The nudging algorithm can be used to find out parameters from the reference data. The method is physics based and easily applicable to different flows.
- Nudging can also be used to learn about the physics of the reference flow.

Nudging is a physics-informed way to probe the equations of motion and determine key degrees of freedom.

Thanks!